Worksheet lecture 19, Confidence Intervals involving proportions, variances, and ratios of variances. Math 481

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1. In a drug trial, the drug was administered to 100 random, independent patients, of which 70 made a full recovery. Find an approximate 90% CI for p, the probability that a patient who gets the drug makes a recovery.

Remark: We are modeling as $X_i =$ the random variable whose value is 1 if patient i recovered, and 0 otherwise. So the population is $Ber(p)$. (The number of people recovered follows $Bin(n,p)$, where n is total number of people)

$$\hat{p} = \frac{70}{100} = .7 \quad Z_{\alpha/2} = 1.645 \quad n = 100$$

$$P\left(.7 - 1.645\sqrt{\frac{.7 \cdot .3}{100}} < p < .7 + 1.645\sqrt{\frac{.7 \cdot .3}{100}}\right) = .9$$

$$P\left(\hat{p} - Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) = 1-\alpha$$

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2. For this question look up tables at the back of the text as needed.

An agricultural company produces two varieties of tomato plants: (1) 'sweet' and (2) 'sour'.

They model that if X is the seasonal yield (in lbs) then X has normal distribution; $N(\mu_1, \sigma_1^2)$ for the first variety and $N(\mu_2, \sigma_2^2)$ for second variety.

In random, independent observations on 2 plants of the first variety, the yields came out to be 6 and 8; and independently, observations with 3 random plants of the second variety, the yields came out to be 6, 7, 5.

(a) Find values of $\overline{X_1}, \overline{X_2}, S_1^2, S_2^2$.

$$\bar{X}_1 = \frac{6+8}{2} = 7 \quad S_1^2 = \frac{(6-7)^2 + (8-7)^2}{2-1} = 2$$

$$\bar{X}_2 = \frac{6+7+5}{3} = 6 \quad S_2^2 = \frac{0^2 + 1^2 + -1^2}{2} = 1$$

(b) Find a 90% CI for the ratio $\frac{\sigma_1^2}{\sigma_2^2}$ of true variabilities in yields of the two varieties.

$$f_{1-\frac{\alpha}{2}, \nu_1, \nu_2} = \frac{1}{f_{\frac{\alpha}{2}, \nu_2, \nu_1}}$$

$$f_{\frac{\alpha}{2}, \nu_2, \nu_1} = 200$$

$$Y = \frac{\sigma_2^2}{\sigma_1^2} \frac{S_1^2}{S_2^2} = 2\sigma_2^2 / \sigma_1^2 \qquad \nu_1 = n_1 - 1 = 1$$

$$\nu_2 = n_2 - 1 = 2$$

$$P(f_{1-\frac{\alpha}{2}, \nu_1, \nu_2} \le Y \le f_{\frac{\alpha}{2}, \nu_1, \nu_2}) = P\left(\frac{2}{f_{\frac{\alpha}{2}, \nu_1, \nu_2}} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{2}{f_{1-\frac{\alpha}{2}, \nu_1, \nu_2}}\right)$$

$$P\left(\frac{2}{18.5} \le \sigma_1^2 / \sigma_2^2 \le 400\right) = .9$$

(c) Find a 90% CI for the true difference in mean yields $\mu_1 - \mu_2$, assuming that the two variances $\sigma_1^2 = \sigma_2^2$.

$$n_1 + n_2 - 1 = 4$$

$$Z = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{S_p \sqrt{1/n_1 + 1/n_2}}$$

$$P(-S_p \sqrt{1/n_1 + 1/n_2}\, t_{\alpha/2, 4} + \bar{X}_1 - \bar{X}_2 < \mu_1 - \mu_2 < S_p \sqrt{1/n_1 + 1/n_2}\, t_{\alpha/2, 4} + \bar{X}_1 - \bar{X}_2)$$

$$P(-S_p \sqrt{1/n_1 + 1/n_2}\, t_{.05, 4} + 1 < \mu_1 - \mu_2 < S_p \sqrt{1/n_1 + 1/n_2}\, t_{.05, 4} + 1) = .9$$

(d) Find a 90% CI for the σ_2^2.

$$\frac{(n_2 - 1) S_2^2}{\sigma_2^2} \sim \chi^2_{n_2 - 1} \qquad P\left(\frac{2}{\chi^2_{.05, 2}} \le \sigma_2^2 \le \frac{2}{\chi^2_{.95, 2}}\right) = .9$$

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